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DARK MATTER, MASS SCALES SEQUENCE, AND SUPERSTRUCTURE IN THE UNIVERSE

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Abstract

There may exist a category of stable non-baryonic dark matter particles in the universe at the present time, they are fermions or bosons with mass $\sim 10^{-1}eV$, which are not in contradiction with the dip phenomena of the extremely high energy primary cosmic ray spectrum at $\sim 10^{15}eV$ ("knee") and $\sim 10^{18}eV$ ("ankle"), and also with the existence of galaxies at large red shift $z \sim 10$. The mass scales sequence connected by a large number A, especially the superstructure scale are helpful for us to understand the Hubble constant and the cosmological constant.

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Up to now, the stable elementary particles existed in nature are nucleon n (mass m_n), electron e, photon γ and neutrino ν, ν can be put into the category of dark matter. For dark matter particles d (mass m_d), they could be fermions f with mass $m_f$or bosons b with mass m_b. Because e and γ make a few contribution to the total mass of the universe at the present time, we shall be confronted with a multi-component $(n+d)$ universe, in which the typical mass scale is the solar mass $M_\odot$. On the other hand, from the fundamental physical constants except electric charge, i.e. the velocity of light c, the gravitation constant G and the Planck constant $\hbar$, a mass scale (Planck mass): $m_{pl} \sim \sqrt{\frac{\hbar c}{G}} \sim 10^{19} GeV$ can be deduced. At first, we discuss the internal relations of $m_{pl}, m_n, m_d, and M_\odot$.

When a star collapses to a neutron star, it can be simplified as a degenerate system composed of neutral nucleons (fermions), in which the boundary momentum of fermions comes to the maximum value $m_n c$. At this time the total number of nucleons in the star with a volume of V is $N = \frac{gVp_o^3}{6\pi^2\hbar^3}$[1], where $g = 2$ and $p_o = m_n c$. The neutron star mass is $M = Nm_n$, and the minimum mass of a black hole (BH) collapsed from a star is $M_{star} \sim M$. Since the classical black hole radius (CBHR) is $r_{star} \sim \frac{GM_{star}}{c^2}$ and the nucleon radius is $r_n \sim \frac{\hbar}{m_n c}$, then

$$M_{star} \sim \frac{m_{pl}^3}{m_n^2} \sim 10^0 M_\odot. \tag{1}$$

It is the scale of the free stream scale (FSS) of nucleons at the early era of the universe. From Eq(1), $\frac{M_{star}}{m_n} \sim (\frac{m_{pl}}{m_n})^3$, and

$$\frac{m_{pl}}{m_n} \sim \sqrt{\frac{\hbar c}{Gm_n^2}} \equiv A. \tag{2}$$

This is the large number used in this paper, $A \sim 10^{19}$. So,

$$\frac{M_{star}}{m_n} \sim A^3, \frac{r_{star}}{r_n} \sim A. \tag{3}$$

From c, G, $\hbar$, a length scale (Planck length) $r_{pl} \sim \sqrt{\frac{\hbar G}{c^3}} \sim \frac{Gm_{pl}}{c^2}$ can be composed, it also has the form of CBHR. Suppose a Planck particle, which mass is m_{pl} and radius is r_{pl}, then

$$\frac{m_n}{m_{pl}} \sim A^{-1}, \frac{r_n}{r_{pl}} \sim A. \tag{4}$$

That is to say, a nucleon can contain $\sim 10^{57}$ Planck particles (string phenomenology)[2] as a star can contain $\sim 10^{57}$ nucleons, but $m_n \ll m_{pl}$, why is it? With the aid of the large number A, the nucleon radius can be expressed in a CBHR form, $r_n \sim \frac{\tilde{G}m_n}{c^2}$, where $\frac{\tilde{G}}{G} = A^2 \sim 10^{38}$, this is just right the ratio of the strong interaction force between two nucleons to the gravitation interaction force between them. So, a nucleon is like a "strong BH" under a "strong gravitation" interaction with a "strong gravitation constant" $\tilde{G}$, and is confined for the "strong signals".

The main results discussed above can be synthesized as follows:

radius	mass	CBHR	FSS
	m_n		
r_{pl}	$m_{pl} = Am_n$	$r_{pl} \sim \frac{Gm_{pl}}{c^2}$	
$r_n = Ar_{pl}$		$r_n \sim \frac{\tilde{G}m_n}{c^2}$	
$r_{star} = A^2 r_{pl}$	$M_{star} = A^3 m_n$	$r_{star} \sim \frac{GM_{star}}{c^2}$	$\frac{m_{pl}^3}{m_n^2} \sim M_{star}$

From this table, one could infer that the next mass scale is $M_F = A^4 m_n \sim 10^{19} M_{\odot}$, that is the superstructure scale in the universe [3–8]. In a $(n+d)$ universe, the scale of M_F may also have a connection with another FSS, i.e. $M_F \sim \frac{m_{pl}^3}{m_d^2}$, since then $m_d \sim A^{-0.5} m_n \sim 10^{-1} eV$. It means that the mass of non-baryonic dark matter particles (NBDMP) is in a $10^{-1} eV$ order of magnitude, and may exist a sequence of mass scales from microcosms to cosmos: $A^{-1.5} m_{pl}, A^{-1} m_{pl}, A^2 m_{pl}, A^3 m_{pl}$, which correspond to the mass scales of dark matter particles, nucleons, stars and the superstructure in the universe respectively.

From now on, we shall directly calculate the mass and the state of NBDMP to check the above deduction about the mass of NBDMP and the sequence of mass scales. If the NBDMP is dominant in the universe at the present time, as a simplification, the direct calculation will be progressed for an one-component universe composed by NBDMP only, and at first one can suppose they are stable and weakly interaction massive fermions (f), i.e. we shall directly calculate the mass m_f, and the state parameters (chemical potential μ_f and temperature T_f) of f-particles using three equations. Under the standard cosmological model and under the nonrelativistic condition, the equation of state is [1]

$$\rho_f = \frac{g m_f^{\frac{5}{2}} (kT_f)^{\frac{3}{2}}}{2^{\frac{1}{2}} \pi^2 \hbar^3} \cdot \int_0^\infty \frac{\sqrt{Z} dZ}{exp(Z-\gamma)+1} = \Omega_f h^2 \rho_c, \tag{5}$$

where g is the variety number of f-particles, $\gamma \equiv \frac{\mu_f}{kT_f}$, and the critical density of the universe is $\rho_c = \frac{3H_{100}^2}{8\pi G}(1+z)^3$, $H_{100} = 100 km \cdot sec^{-1} \cdot Mpc^{-1}$, $h = \frac{H_o}{H_{100}}$. z is red shift. The evolutionary equation of temperature is

$$T_f \doteq \frac{kT_{fo}T_{\gamma o}}{\xi m_f c^2} \cdot (1+z)^2 = \frac{k\tilde{T}_{fo}T_{\gamma o}}{m_f c^2}, \tag{6}$$

in which $T_{\gamma o}$ is the microwave background temperature, $T_{\gamma o} = 2.7K$; T_{fo} is the f-particles temperature when $m_f = 0$; $\tilde{T}_{fo} = \frac{T_{fo}}{\xi} \cdot (1+z)^2$, ξ is a phenomenological parameter reflected non-relativity, $\frac{kT_{\gamma o}}{m_f c^2} \leq \xi \leq 1$. The third equation is in relation to the superstructure of the universe mentioned above. Since last decade some reports related to the very large scale structure (superstructure) in the universe are published [3−7], especially the reports about the periodic superstructure [6−7] enlighten us that the formation of such structure may relate to the gravitation and the hydrodynamic effect in cosmic medium, because the scale of the superstructure has been $1\% - 10\%$ of the present horizon, it will be as well to adopt that the sound velocity v_s in cosmic medium is $v_s = 0.01c - 0.1c$.

$$v_s = \sqrt{\frac{10}{9}\frac{kT_f}{m_f} \cdot \int_0^\infty \frac{Z^{\frac{3}{2}} dZ}{exp(Z-\gamma)+1} / \int_0^\infty \frac{\sqrt{Z} dZ}{exp(Z-\gamma)+1}} \sim 0.01c - 0.1c. \tag{7}$$

Since then the results of

$$\begin{aligned} &v_s = 0.01c - 0.1c \\ &m_f = 10^{-1} - 10^{-2} eV \\ &\mu_f = 10^{-5} - 10^{-4} eV \\ &\xi T_f = 10^{-3} - 10^{-2}\ {}^oK \\ &\gamma = 10^1 - 10^2 \end{aligned}$$

are obtained [9] for $z = 0$ and $w \equiv \frac{g}{\Omega_f h^2} = 1 \sim 80$. From the values of γ, it means that the f-particles are in a degenerate state. Under the degenerate approximation, we have $m_f^4 = \frac{2\pi^2}{3^{\frac{1}{2}}} \cdot \frac{\hbar^3 \rho_c}{w v_s^3}$, $\mu_f = \frac{3}{2} m_f v_s^2$; $\frac{T_f}{\tilde{T}_{fo}} \doteq \frac{kT_{\gamma o}}{m_f c^2}$. So, the values of m_f has nothing to do with z, and is not sensitive to the parameters g, Ω_f, h ($m_f \propto w^{-\frac{1}{4}}$).

Because there exists some periodic superstructure in the universe [6]−[7] and the Jeans length $\lambda_J \sim \frac{v_s}{\sqrt{G\rho_f}} \sim 10^2 Mpc$[6], the concrete value of $\frac{v_s}{c}$ at z=0 in a equivalent homogeneous universe is: $\frac{v_s}{c} \sim \frac{\lambda_J}{r_H} \sim 0.01$, where r_H is the present horizon. From the above results of calculation, m_f is $\sim 10^{-1} eV$ indeed.

But, the maximum scale of superstructure from observations is $\sim 10^3 Mpc$[6], it corresponds to a typical mass scale M_F mentioned above and will appear during the H-decoupling when $m_f \sim 10^{-1} eV$. Once a superstructure broke away from the cosmic expand, the celestial bodies with different scales originated from various cosmic perturbations were speedily produced in such region [8] as in a quasi-static universe. One of the essential conditions is that the particles of cosmic medium in M_F region must be in a very non-relativistic state with a average thermal velocity $\bar{v} \sim v_s \sim 0.1c$. From Eq(6) and Eq(7), we know this time is $z \sim 10$, and may be near by the time that superstructure broke away from cosmic expand. So, the existence of stable NBDMP with mass $\sim 10^{-1} eV$ is not in contradiction with the recent report about the existence of galaxies at large red shift $z \sim 10$[10].

Another way to calculate the matter state of f-particle is that the evolutionary equation of temperature, Eq(6), is substituted by the concrete chemical potential value of the f-particle. According to the isoentropic hypothesis in the evolution of the universe, the entropy per f-particle (S/N) is a constant:

$$
\begin{aligned}
S/N \;=\; & \frac{k}{3} \cdot \{ \int_0^\infty Z^{\frac{3}{2}} (Z + \frac{2m_f c^2}{kT_f})^{\frac{3}{2}} \frac{dZ}{exp(Z-\gamma)+1} \\
& +3 \int_0^\infty (Z + \frac{m_f c^2}{kT_f}) \sqrt{Z(Z + \frac{2m_f c^2}{kT_f})} \frac{(Z-\gamma)dZ}{exp(Z-\gamma)+1} \} \div \\
& \int_0^\infty \frac{dZ}{exp(Z-\gamma)+1} \cdot (Z + \frac{m_f c^2}{kT_f}) \sqrt{Z(Z + \frac{2m_f c^2}{kT_f})}.
\end{aligned}
$$

Under the relativistic condition, $S/N = \frac{k}{3} \cdot \frac{4J_3 - 3J_2\gamma}{J_2}$, but, under the non-relativistic condition, $S/N = \frac{k}{3} \cdot \frac{5J_{\frac{3}{2}} - 3J_{\frac{1}{2}}\gamma}{J_{\frac{1}{2}}}$, where $J_a \equiv \int_0^{\infty} \frac{Z^a dZ}{exp(Z-\gamma)+1}$. On the other hand, under the relativistic condition there are two situations for μ_f[11] typically: the first is $\gamma > 20$(degenerate state) and the second is $\gamma = 0$. From calculations we know for $\gamma > 20$ the non-relativistic fermions deduced from relativistic fermions will still be near by a degenerate state; but for $\gamma = 0$, the value of γ for non-relativistic fermions will not still be equal to zero but will translate to $\gamma = -1.62$ after the deducing. We can substitute such value of γ for the evolutionary equation of temperature and obtain the approximate expressions of m_f, T_f, and μ_f: $m_f^4 = (\frac{10\pi}{3})^{\frac{3}{2}} \frac{\hbar^3 \rho_c}{w v_s^3 \cdot exp(\gamma)}$, $T_f = \frac{3}{5} \frac{m_f v_s^2}{k}$, $\mu_f = \frac{3\gamma}{5} m_f v_s^2$. On the basis of the value of $\gamma = -1.62$ and the parameter ranges as before, the calculated values of m_f and $\mid \mu_f \mid$ are approximately unchanged.

If the NBDMP are bosons, since the chemical potential of bosons are minus, all of the approximate equations and results for f-particles with minus chemical potential are still suitable to b-particles, but the subscript f must be substituted by b, and the term $[exp(Z-\gamma)+1]$ must be substituted by $[exp(Z-\gamma)-1]$. When $\mu_b = 0$, for b-type dark matter particles, they will have a minimum mass $m_b = (4.77 \frac{\hbar^3 \rho_c}{w v_s^3})^{\frac{1}{4}} \sim 10^{-1} eV$, and would be different from the ordinary axions.

In summary: (1) There may exist a category of stable NBDMP in the universe at the present time, which relate to the superstructure of the universe in due time. These particles are fermions or bosons. In both cases, we deduce that the particle mass is $\sim 10^{-1} eV$ and the absolute value of its chemical potential is $\ll 10^{-1} eV$. These results are not in contradiction with the existence of galaxies at large red shift $z \sim 10$ and with the dip phenomena of the extremely high energy primary cosmic ray spectrum at $\sim 10^{15} eV$ ("knee") corresponding to fermion NBDMP and at $\sim 10^{18} eV$ ("ankle") corresponding to boson NBDMP. (2) This paper is consistent with our previous works [8],[12],[13]. If the NBDMP with mass $\sim 10^{-1} eV$ do exist in the universe, they can be used to explain the large scale stream [8] and the filament [12] in the universe, and also to explain the flatness of the rotational velocity distribution in spiral galaxies [13]. (3) If the f-particles are neutrinos, since the value of m_f is not sensitive to parameter Ω_f, the neutrino mass is also $\sim 10^{-1} eV$[14]. (4) If the superstructure scale M_F indeed exists in the universe, on this basis is the true cosmology principle, so, for the observed value of the Hubble constant H_o and the cosmological constant Λ, it must be considered the influence of the superstructure M_F. That is to say, the value of Λ from the data about the SNe Ia [15] could still be equal to zero [16]. (5) The concept of large number was introduced by P.A.M.Dirac [17]. In this paper the large number A connects microcosms with cosmos by a sequence of mass scales, and also contributes to probe the precise structure of nucleon [2]. (6) Under the framework of this paper, there is no room for stable NBDMP with heavy mass dominated in the universe at the present time. If they are existence in the hole region of our galaxy by a violent relaxation process, why our galaxy is a spiral galaxy and not is a elliptical galaxy? Once these heavy particles (may be SUSY particles) are not recognized in the experiments during next decade as the present status about 17 keV neutrinos or monopoles, the NBDMP discussed in this paper and the cold universe will be progressively researched again [18].

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